MODELLING OF REINFORCED CONCRETE SLAB TO ACCOUNT FOR CRACKING

Taliat Azizov[1], Nataliia Sribniak[2], Liudmyla Tsyhanenko[2], Dmytro Volkov[2]
[1]Pavlo Tychyna Uman State Pedagogical University, Ukraine;
[2]Sumy National Agrarian University, Ukraine
taljat999@gmail.com, nataliya.sribnyak.17@gmail.com, tsyganenkola@ukr.net,
volkovvdg@gmail.com

Abstract. Slab-ribbed and slab-plate reinforced concrete floor systems, which are statically indeterminate systems, mostly operate in the plastic stage of the material. This stage of the element under load is characterized by cracking, and the deflections of the system are much larger than the analysis of the system in elastic behaviour shows. The presence of cracks determines redistribution of the stiffness of floor system elements. This fact, in turn, causes redistribution and change in the values of both bending and torque moments. The creation of a correct finite element model will allow to conduct a numerical experiment to study the effect of normal cracks on changes and redistribution of internal forces in the elements of slab-ribbed and slab-plate floor systems. The creation of a particular finite element model by certain methods, as well as the use of certain finite elements (rod, plate, volume), is primarily determined by the specific analysis task and its goals. The article describes the finite element rod model of multi-cavity slabs, formulates the algorithm of the numerical experiment, and conducts the numerical experiment. The results of the study allow us to practically confirm the theoretical predictions regarding the importance of taking into account not only bending stiffness but also torsional stiffness for slab-beam and slab-plate reinforced concrete systems in the analysis. The conclusions show that the iterative analysis, which takes into account the gradual appearance of normal cracks due to the redistribution of bending moments in a spatially statically indeterminate system (a cell of multi-cavity plates), makes it possible to estimate the real stress-strain state in its elements. An important fact has been established that the torques increase only in the elements that are not cracked and decrease several times in other elements.

Keywords: plate, rod, stiffness, torsion, crack, finite element.

Introduction

The aim of the research is to determine and evaluate the stress-strain state of a finite element scheme of a monolithic ribbed (caisson) reinforced concrete slab with consideration of normal crack formation. The object of study is a monolithic caisson slab with a plan size of 12.0x12.0 m. This study is relevant for both new design and reconstruction of reinforced concrete structures [1].

Various factors, such as local loading of the reinforced concrete slab disc, arrangement of openings in the structure, asymmetric loading, etc. cause not only bending moments but also significant torques to occur in the elements of such slab [2-10]. Even after the elastic analysis of the system (without taking into account the occurrence of normal cracks), it can be stated that significant torques occur in the floor elements. Obviously, a decrease in stiffness in both torsion and bending will redistribute the torque and bending moments.

Studies have shown [4; 11-13] that the torque values can increase significantly, many times, from the torque values obtained by elastic analysis.

The redistribution of forces in the plastic stage of behaviour of such systems is significantly influenced by cracks of different types and different nature of formation. Normal cracks (perpendicular to the longitudinal axis of the element) are the most typical for slab-beam flooring systems. They are caused by the bending moment.

Figure 1 shows a photo of a reinforced concrete bridge beam as an example of a structure operating in a complex stress state. On the faces of such a bridge beam, normal cracks can be clearly distinguished precisely from the bending action. Papers [3; 14-17] show the importance of taking into account the effect of crack formation on the torsional stiffness of reinforced concrete elements. However, these works consider only spatial (spiral) cracks. In this regard, during the analysis of reinforced concrete rod systems, as a rule, only changes in bending stiffnesses caused by the formation of normal cracks are taken into account. The change in torsional stiffness is neglected. However, as shown in [4; 5; 11-13; 18], normal cracks also significantly affect the change in torsional stiffness of reinforced concrete elements. Neglecting the change in torsional stiffnesses due to the formation of normal cracks leads to significant errors in the determination of deflections and forces in reinforced concrete cross-beam systems and solid slabs.

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Fig. 1. **Normal bending cracks (1) in reinforced concrete bridge beams (Uman, Ukraine)**

Therefore, determining the influence of changes in torsional stiffness due to the formation of normal cracks on the redistribution of forces and displacements in reinforced concrete statically indeterminate systems is an important task of civil engineering science.

Thus, it can be stated that:

- in reinforced concrete slab and beam floor systems, under certain conditions, torques reach significant values;
- in reinforced concrete slab-beam floor systems, which are statically indeterminate, a significant percentage of all possible cracks are perpendicular to the longitudinal axis of the element (normal) cracks;
- occurrence of normal cracks causes weakening of the element cross-section and a decrease in both bending and torsional stiffness;
- the change (redistribution) of stiffnesses due to cracking causes a change in the maximum values and nature of bending, torque diagrams, and deflections;
- for a more accurate and correct static analysis of the slab, the effect of normal crack formation on the change in torsional stiffness should be taken into account.

Materials and methods

To perform the numerical experiment proposed in this research, we used the finite element method in the Lira-SAPR software package. The results of the static analysis of the plate finite element scheme, which can be used to model the caisson slab, are line bending and torsional moments. But the difficulty is in assigning modified (reduced) bending and torsional stiffness due to the formation of normal cracks. Therefore, this modelling method is not sufficiently convenient for the purpose of the study.

Let us consider another method of analysis of a monolithic reinforced concrete slab – the method of approximation by a system of cross bars. This method was proposed in [13; 19; 20]. The cross-rod system approximation method can be applied to any solid element (structure). The application of the formulas of the theory of elasticity can be justified only in analysis of the elastic stage of the element behaviour and is not applicable to the analysis taking into account cracking [5; 10; 11], which occurs exactly in the plastic stage of its behaviour. Paper [19] shows that it is convenient to use the rod approximation method when analysing reinforced concrete slabs (solid, multi-cavity, ribbed) with regard to cracking. The possibility of such approximation is due to the fact that the influence of cracks of different types both on bending stiffness and torsional stiffness for reinforced concrete bar elements is sufficiently studied. In the study [13; 19] it is also shown that a slab of solid section can be analyzed by approximating it with a cross-bar system. In the studies by A.R. Rzhanitsyn, an approximation of solid slabs structures as longitudinal, transverse and diagonal rods without taking into account their torsional stiffness is proposed. In [19; 20] it is proposed to approximate a plate structure by a system of only mutually perpendicular longitudinal and transverse rods. The torsional stiffness of these rods is taken into account. In [19; 20], the method of rod approximation of structures of solid slabs supported along the contour is presented. A method for determining the torsional stiffnesses for the rods used to approximate such a slab is also presented.

To study the influence of change in torsional stiffnesses of elements from cracking, we consider a caisson slab. In this case, we apply a rod approximation of the slab ribs using a rod finite element of spatial orientation with assignment of stiffness characteristics. In the Lira-SAPR software package, this is finite element No 10.

Main part

The slab has a plan dimension of 12.0 × 12.0 m (Fig. 2, b). The cross-section of the ribs is the same in both directions (Fig. 2, a).

In the Lira-SAPR 2024 software package, we will generate a finite element model of a monolithic caisson slab (Fig. 3).

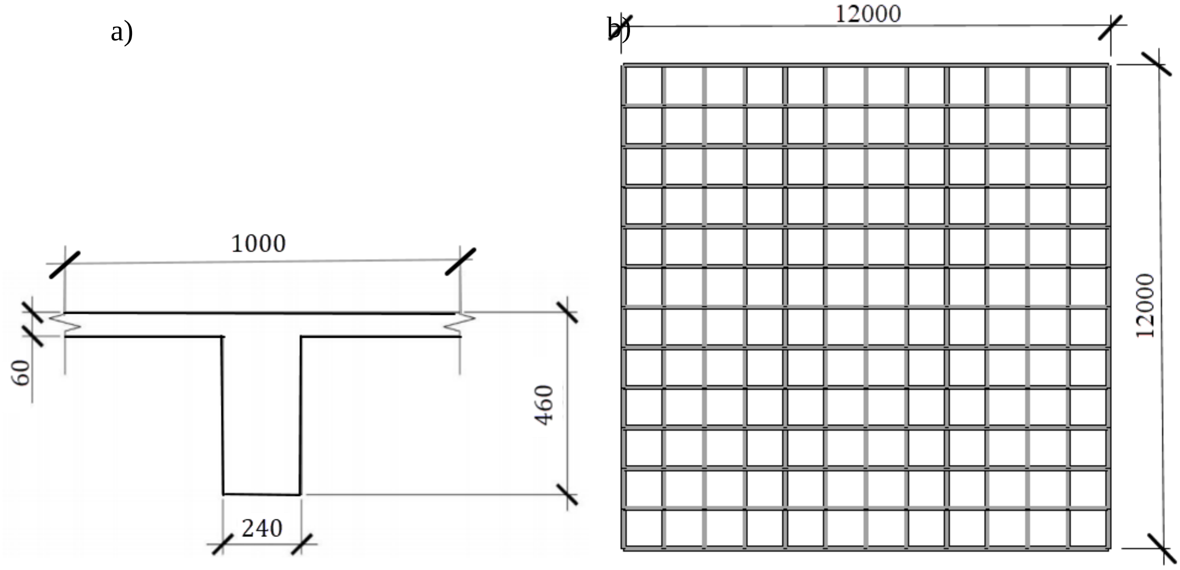


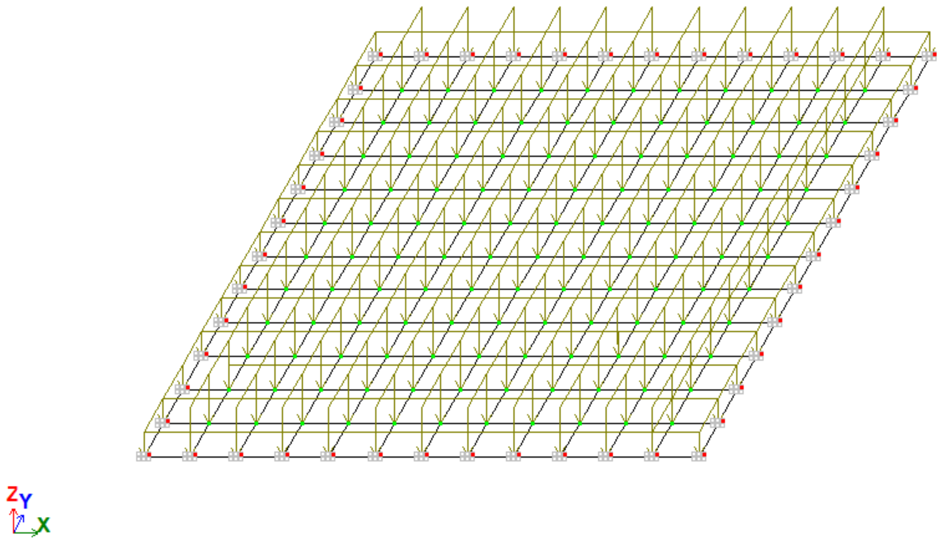
Fig. 2. **Structural design of the slab:** a − cross-section of reinforced concrete caisson slab; b − 3D model of the ribs of the caisson slab in the plane of XOY in the Lira SAPR 2024 software package

The slab is made of concrete of class C20/25 with the following characteristics [21]: f_{cd} = 14.5 MPa, $f_{ctk,0.05}$ = 1.5 MPa = 0.15 kN·cm^{-2}, E_{cd} = 23 GPa = 2.3·10^4 MPa = 2300 kN·cm^{-2}.

The scheme in Fig. 2,b was modelled using rod finite elements (FE #10 - universal spatial rod). The slab part of the floor is not symbolically shown, but its weight is taken into account in the external load on the ribs. The boundary conditions of the structure are shown in Fig. 3. Thus, only the linear movement of the nodes of the finite element scheme along the vertical Z is prohibited.

The reinforced concrete slab is hingedly supported along the perimeter (linear movement along the Z axis is prohibited). The load acting uniformly over the slab area is 0.009 MPa (9.0 kN·m^{-2}) and is reduced to a linearly distributed load along the length of the rib. The reduced load is uniformly distributed along the length of the bar and is 9.0 kN·m^{-1} for the middle ribs and 4.5 kN·m^{-1} on the outer ribs. The numerical description of the stiffness characteristics for the T-section according to Fig. 2.a for the rods that are modelled by the FE model is adopted. This way of setting the stiffness parameters in the software package was adopted taking into account that the numerical parameters can be easily changed, reducing the stiffness in the analyses. At the first stage (Fig. 4) it was planned to do static analysis of the system, the elements of which were assigned elastic stiffness values. Then we calculated the cracking moment M_{crc} and compared the bending moment M_y at the sections of the bars of the finite element scheme according to Fig. 3. The elements where the maximum bending moment $M_{max} > M_{crc}$, appears normal from bending cracks. In the slab scheme, these sections were inverted (Fig. 5) to further modify their stiffness characteristics.

a)



b)

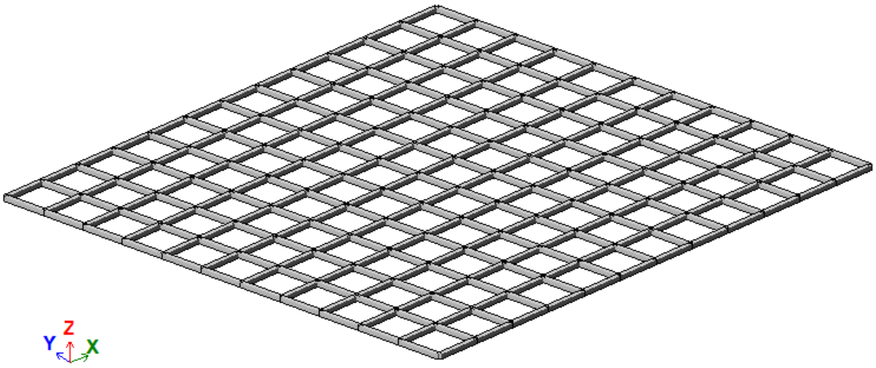


Fig. 3. **Model accepted for analysis:** a − finite element model of the caisson slab according to Fig. 2; b − isometric projection of the spatial model of the ribs

At the next stages, iterative analyses were performed in two variants: with only one bending stiffness $EI_y/4$ reduced by 4 times and with both bending and rotation stiffnesses $EI_y/4$ and $GI_t/4$ reduced by 4 times, too. A fourfold decrease in bending stiffness with the formation of normal cracks is accepted according to the research [4] as for medium-reinforced concrete elements. The decreasing of torsional stiffness is assumed to be the same as for bending stiffness according to the data of rod approximation of solid slabs [13].

Iterative analyses were ended when the stiffness value of the element with modified stiffnesses in the last iteration was the same as the stiffness value of the elements in the previous iteration. The modified stiffness type is a reduced stiffness type because it took into account cracking. The number of iterations was usually 5...6 iterative analyses.

Cracking reduces the value of both bending and torsional stiffnesses of the rod cross-section. It is logical to predict that the bending stiffness will decrease significantly. But many both experimental and theoretical researches show that when normal cracks occur in floor beam elements, the torsional stiffness of such elements also changes significantly [5-9; 11]. Therefore, it is an actual task to research the degree of change in the torsional stiffness of these rods.

According to [21] for concrete of class C20/25, the characteristic tensile strength of concrete is as follows $f_{ctk,0.05}$ = 1.5 MPa = 0.15 kN·cm^{-2}.

Determination of the value of the bending moment that causes cracking:

$$M_{crc} = f_{ctk} \cdot W_{pl}, \tag{1}$$

where M_{crc} – value of the bending moment at which normal cracks are formed, kN·m;
f_{ctk} – characteristic tensile strength of concrete, MPa;
W_{pl} – moment of resistance of the rod cross-section when the element is operating in the plastic stage of deformation, cm^3.

$$W_{pl} = W_{elast} \cdot 1.5, \quad (2)$$

where W_{elast} – moment of resistance of the cross-section W_{pl} during operation in the plastic stage, cm^3.

For the section of the rod according to Fig. 2, a, the value of plastic moment of resistance of the T-section section is W_{pl} = 19000 cm^3. In iterative analyses at step 6 and 8, the numerical method of describing the stiffnesses for the T-section was used (Fig. 2 a), but the values of elastic stiffnesses EI_y and GI_t were reduced by a factor of 4.

Fig. 4 shows the algorithm of the numerical experiment.

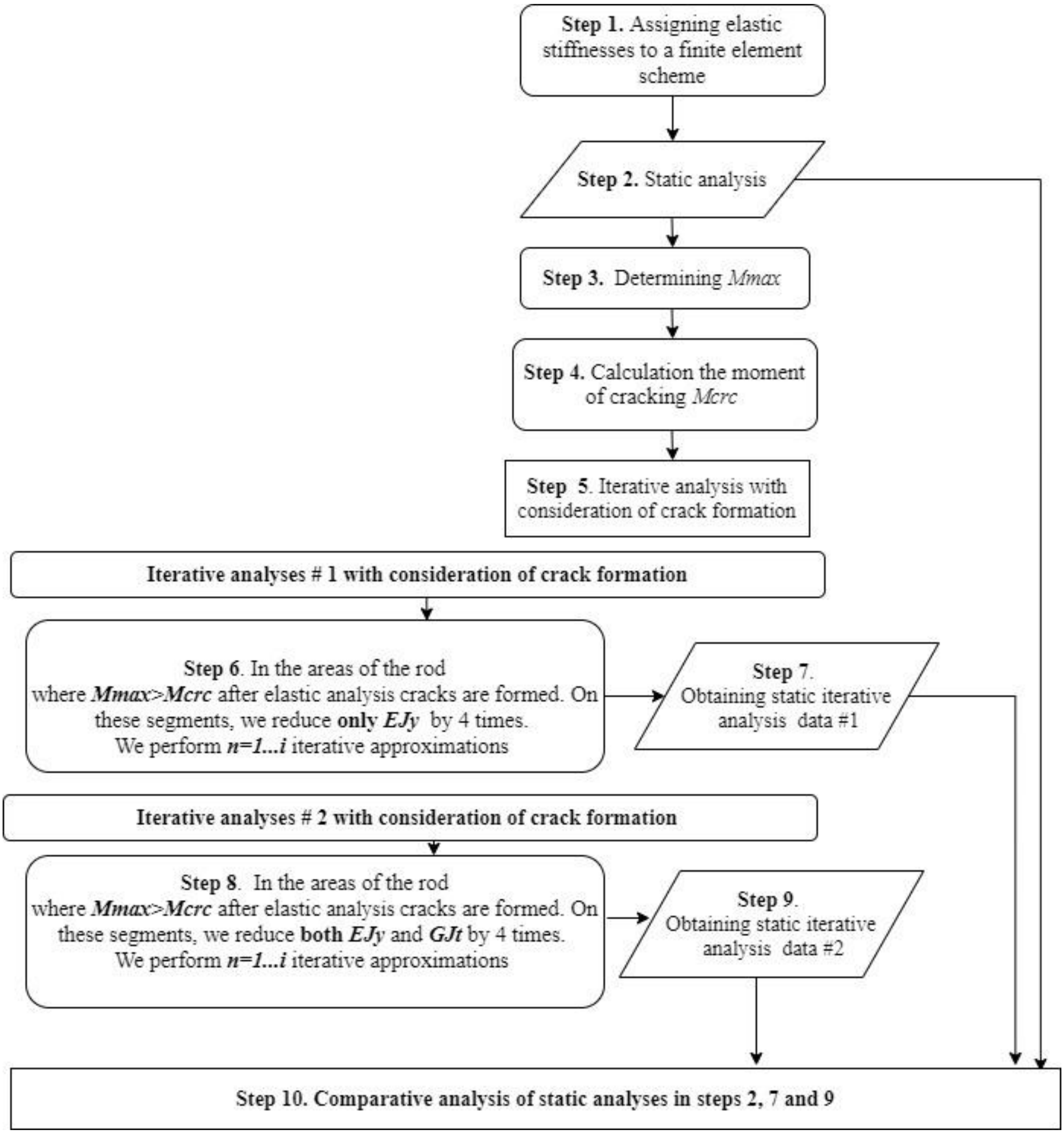


Fig. 4. **Algorithm of the numerical experiment**

Thus, when analysing the moment diagram $M_{y,\,elast}$ (Fig. 5), it is possible to determine the parts of the rods in the cross-sections where the maximum bending moment $M_{y,\max}$ from the external load will exceed M_{crc}.

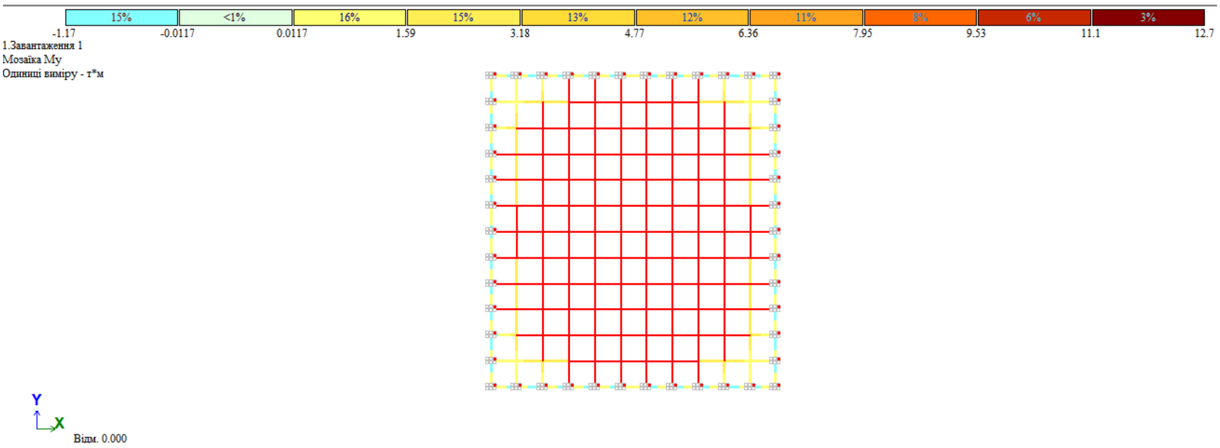


Fig. 5. **Sections of the rod diagram (inverted in red), where after the elastic analysis $M_{y,max} \geq M_{crc}$ normal cracks from bending arise**

Results and discussion

The maximum displacements and bending moments after static analyses are given in Tables 1, 2.

Table 1

Maximum vertical movements f_z of nodes

$f_{z,max}$, mm				
Analysis with elastic stiffnesses	**Reduced only bending stiffness by a factor of 4**	**Reduced both bending and torsional stiffness by a factor of 4**	**(Data of column "1"/ Data of column "2") × 100%**	**(Data of column "1"/ Data of column "3") × 100%**
1	2	3	4	5
14.2	45.7	55.1	31.07	55.80

Table 2

Maximum bending moments $M_{y,\,max}$ and maximum torques $M_{t,\,max}$

Analysis with elastic stiffnesses	**Reduced only bending stiffness by a factor of 4**	**Reduced both bending and torsional stiffness by a factor of 4**	**(Data of column "1"/ Data of column "2") × 100%**	**(Data of column "1"/ Data of column "3") × 100%**
1	2	3	4	5
$M_{y,\,max}$, kN·m				
127.13	90.56	124.5	40.4	2.11
$M_{t,\,max}$, kN·m				
17.65	49.3	42.8	35.8	41.2

Figure 6 shows the deflection diagrams in the middle line of the overlap. It shows that when only the bending stiffness changes due to cracking (the traditional design concept) are taken into account, the maximum deflection increases by a factor of 3.2. Taking into account changes in both bending and torsional stiffnesses increases the maximum deflection by 3.9 times.

Thus, the value of the maximum deflection when taking into account changes in both bending and torsional stiffnesses is 20.6% higher than the maximum deflection when taking into account changes in bending stiffnesses only.

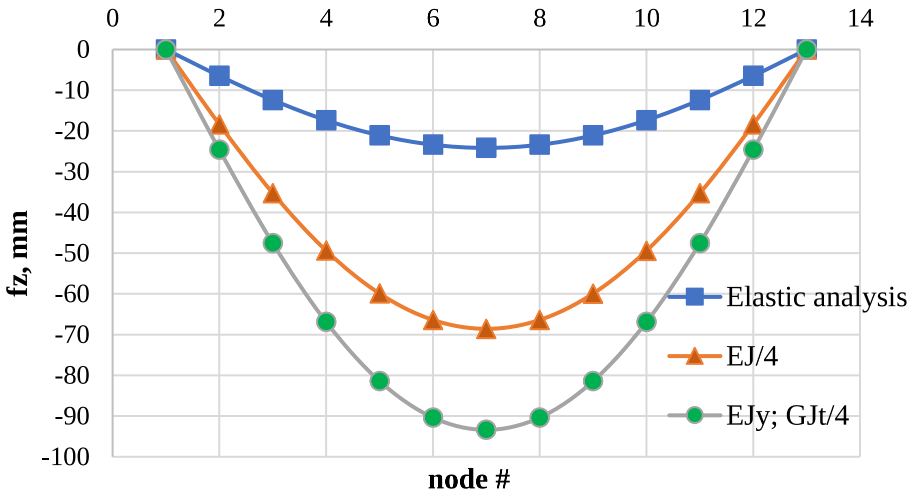


Fig. 6. **Deflection diagram in the middle of the overlapping lines (displacement of nodes f_z)**

The difference in bending and torque values is shown in Fig, 7-8.

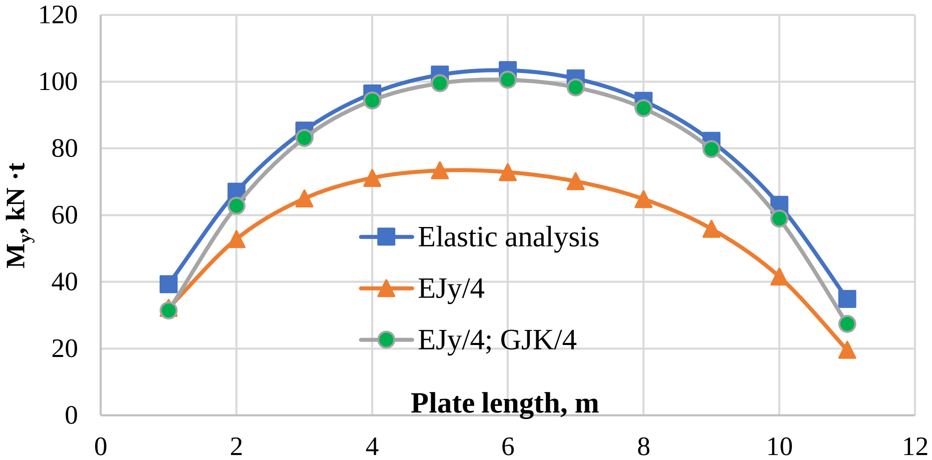


Fig. 7. **Bending moment diagram M_y in the most stressed slab rib by different types of analysis**

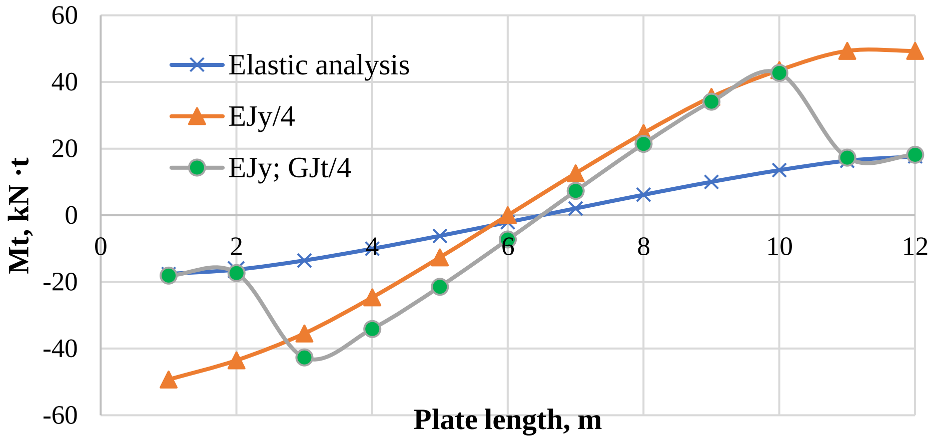


Fig. 8. **Diagrams of torques M_t in the most stressed slab rib by different types of analysis**

An important conclusion can be made from Tables 1 and 2, as well as Fig. 6-8, that taking into account changes in torsional stiffness significantly changes both the displacement pattern and the pattern of bending and torsional moments in the elements of the considered slab. Figure 6 also explains why, in practical design, the actual deflections of reinforced concrete floor slabs are often greater than the analytical deflections. This is exactly because in traditional design designers only consider the changes in bending stiffnesses from cracking. This not only distorts the displacement pattern, but also the bending moment and torque pattern.

The finite element design model, first and foremost, depends on the task to be solved with the help of the FE model. For example, there are many ways to model ribbed slabs, including their variant, caisson slabs. As a rule, ribbed slabs can be modelled in several ways:

- the slab part is modelled by plate finite elements, and the ribs are modelled by rod finite elements (in this case, the “ absolutely rigid body” or “ absolutely rigid segment” tools are used to reflect the eccentricity of the joint between the slab and rib parts [22]);
- the plate part is modelled by plate finite elements and the ribbed part is also modelled by plate finite elements;
- both the plate and the ribbed parts can be modelled by volumetric finite elements.

Since the purpose of the study is to identify changes in the forces − bending moment, torque - occurring in the beams (ribs of the caisson slab), the model of such a slab with the representation of the ribs as rods is the most adequate and meets the goal.

Representation of the slab ribs in the form of rod elements allows taking into account the change in the bending and torsional stiffness of the elements of this slab.

The proposed model makes it possible to see the difference in forces and displacements of the rods of the analysed system depending on the influence of cracks on torsional stiffness or without such influence.

As a result of the research, it is concluded that the design of cross-rod systems and slabs should take into account the change in torsional stiffness of reinforced concrete elements from cracking. Otherwise, it leads to incorrect determination of forces and displacements.

The issue under study is important in view of the following. Changes in torsional stiffness from the formation of normal cracks are not provided for in any software package (Ukraine, EU, USA). The paper proposes and proves that ignoring this factor leads to errors in the computation of displacement and forces, especially torques.

Conclusions

1. The reinforced concrete caisson slab was modelled as a cross-rod system, which allowed to take into account the change in bending and torsional stiffness of the elements of this slab.
2. The redistribution of internal forces in the bars of the finite element model as a result of cracking has been taken into account, which allowed us to estimate the real stress state in its elements, taking into account changes in both torsional and bending stiffnesses of the slab elements.
3. The difference in analyses when only the bending stiffness of the cross-section is reduced as a result of cracking compared to analyses when both bending and torsional stiffnesses are changed is: for deflections − 20.6%; for bending moments − 37.5%; for torsional moments − 15.2%.
4. When designing reinforced concrete slabs, both solid and rod slabs, it is mandatory to take into account the change in both types of stiffnesses of their elements from the formation of normal cracks in them.

Author contributions

Conceptualization and methodology, T.A.; investigation, T.A., N.S. and L.Ts.; writing − original draft preparation, T.A. and N.S.; writing − review and editing, D.V. All authors have read and agreed to the published version of the manuscript.

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